



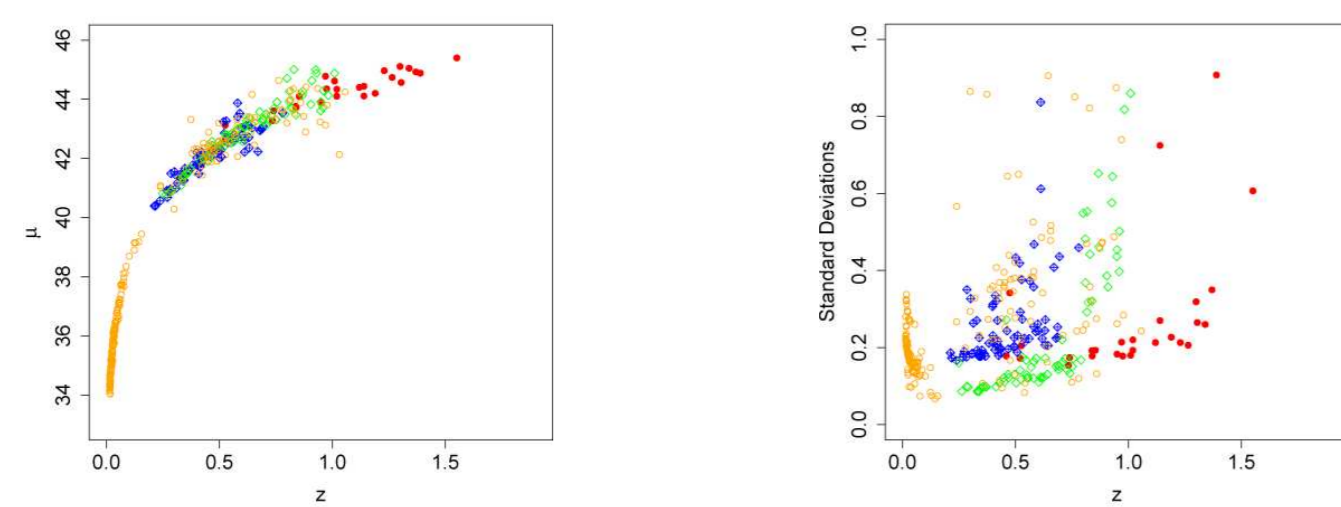
Abstract

The fact that the Universe is expanding has been known since the 1920's. If the Universe was filled with ordinary matter, the expansion should be decelerating. Beginning in 1998, however, observational evidence has been accumulating in favor of an accelerating expansion of the Universe. The unknown driver of the acceleration has been termed dark energy. The nature of dark energy can be investigated by studying its equation of state, that is the relationship of its pressure to its density. The equation of state can be measured via a study of the luminosity distance-redshift relation for supernovae and can be further constrained by adding baryon acoustic oscillation (BAO) and cosmic microwave background data (CMB). In this study, we employ supernovae data, BAO, and CMB data, including measurement errors, to determine whether the equation of state is constant or not. Our method is based on Bayesian analysis of a differential equation and modeling $w(z)$ directly, where $w(z)$ is the equation of state parameter. This work stems from collaboration between UCSC and Los Alamos National Laboratory (LANL) in the context of the Institute for Scalable Scientific Data Management (ISSDM) project.

Data

Supernovae Data (SNe)

There are 397 SNe in this SALT data set. Each SNe has a redshift (z) value, and observed distance modulus (μ), uncertainty measure for μ (τ). The plots are colored by the telescope or observational group.



Supernovae will have the following likelihood equation in our analysis:

$$L \propto \left(\frac{1}{\sigma}\right)^n e^{-\frac{1}{2} \sum \left(\frac{\mu_i - T_1(z_i, H_0, \Omega_m, w(u))}{\tau_i \sigma} \right)^2}$$

The main parameter of interest is $w(u)$ (Dark Energy equation of state). There are four other unknown parameters: the variability (σ^2), a scaling constant (M), the dimensionless matter density parameter (Ω_m), and Hubble's parameter (H_0), also $\Omega_r = 0.247/H_0^2$ and $c = 3 * 10^5$ is the speed of light. The data is given as z_i , μ_i , and τ_i , so these are known values in our equations.

$$T_1(z, ...) = M + 25 + 5 \log_{10} c (1 + z_i) \int_0^{z_i} \frac{1}{H_0 h(s)} ds$$

$$h(s) = \left(\Omega_r (1+s)^4 + \Omega_m (1+s)^3 + (1 - \Omega_m - \Omega_r) (1+s)^3 e^{-3 \int_0^s \frac{-w(u)}{1+u} du} \right)^{1/2}$$

BAO

BAO data will have the following likelihood equation in our analysis:

$$L \propto \left(\frac{1}{\sigma}\right)^n e^{-\frac{1}{2} \sum \left(\frac{.469 \left(\frac{n_s}{.98} \right)^{-.35} - T_2}{0.017\sigma} \right)^2}$$

Currently, there is only one measured value available for BAO: $z_1 = 0.35$ where $A = .469 \left(\frac{n_s}{.98} \right)^{-.35} \pm 0.017$ and $n_s = 0.958 \pm 0.016$. We will use a hierarchical form to account for both types of uncertainty in A . n_s will be treated as a parameter with prior $N(0.958, 0.016^2)$. The data relation transform in this case will be:

$$T_2 = \frac{\sqrt{\Omega_m}}{h(z_1)^{1/3}} \left(\frac{1}{z_1} \int_0^{z_1} \frac{1}{h(s)} ds \right)^{2/3}$$

CMB

CMB data will have the following likelihood equation in our analysis:

$$L \propto \left(\frac{1}{\sigma}\right)^n e^{-\frac{1}{2} \sum \left(\frac{R - T_3(z_2, H_0, \Omega_m, w(u))}{0.020\sigma} \right)^2}$$

The CMB data comes from the Wilkinson Microwave Anisotropy Probe: $R = 1.713 \pm 0.020$ at redshift $z_2 = 1087.86 \pm 1.13$. $T_3 = \sqrt{\Omega_m} \int_0^{z_2} \frac{1}{h(s)} ds$

Models

Parametric

Model 1 - $w(u) = a$, where a is a constant and Model 2 - $w(u) = a + b \left(\frac{1}{1+u} - 1 \right)$

With these models the inner integral can be done analytically. Unfortunately, these models are not very flexible. It is hard to quantify how they fit $w(u)$ because it is a second derivative of the observed data and residual plots and goodness of fit tests tend to be inaccurate. (Priors are needed for all models $\pi(\sigma^2) \sim IG(10, 9)$, $\pi(H_0) \sim N(71.9, 2.7)$, $\pi(\Omega_m|H_0) \sim N(0.1326(100/H_0)^2, (0.0063(100/H_0)^2)^2)$, $\pi(a) \sim U(-25, 1)$, and $\pi(b) \sim U(-10, 10)$,

Non-parametric

Model 3 - Gaussian Process Model

$w(u) \sim GP(-1, \Sigma_{22} = \kappa^2 K(u, u'))$ where $K(u, u') = \rho^{|u-u'|}$ is the exponential correlation function. The inner integral in this case cannot be done analytically but has the property that $y(s) = \int_0^s \frac{w(u)}{1+u} du$ is also a GP that can be found by integrating the correlation function of $w(u)$.

$$y(s) \sim GP(-\ln(1+s), \Sigma_{11} \kappa^2 \int_0^s \int_0^{s'} \frac{\rho^{|u-u'|}}{(1+u)(1+u')} du du')$$

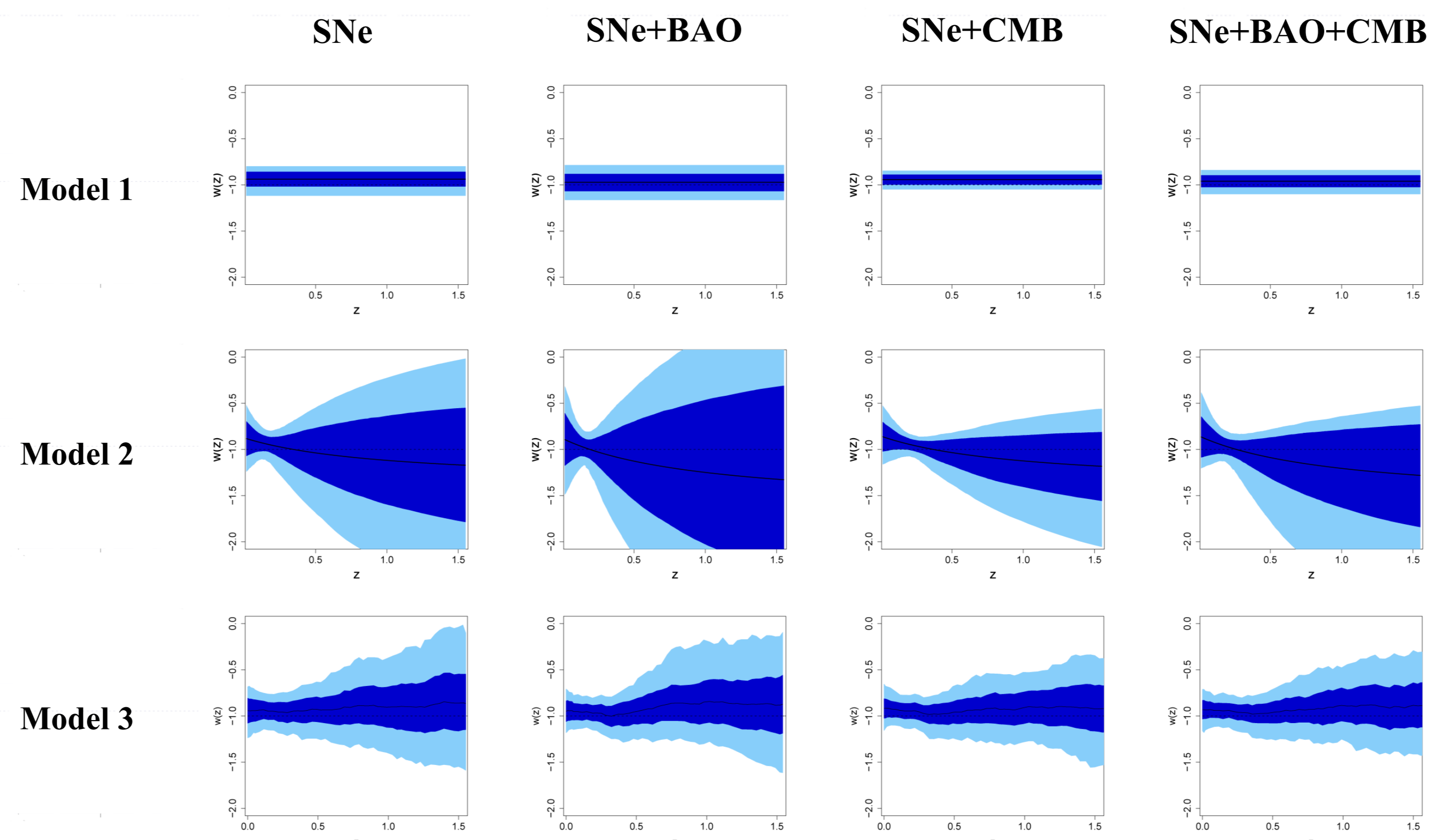
We will use the mean as our inner integral:

$$E[y(s)|w(u)] = -\ln(1+s) + \Sigma_{12} \Sigma_{22}^{-1} (w(u) - (-1)) \text{ where } \Sigma_{12} = \kappa^2 \int_0^s \frac{\rho^{|u-u'|}}{(1+u)} du$$

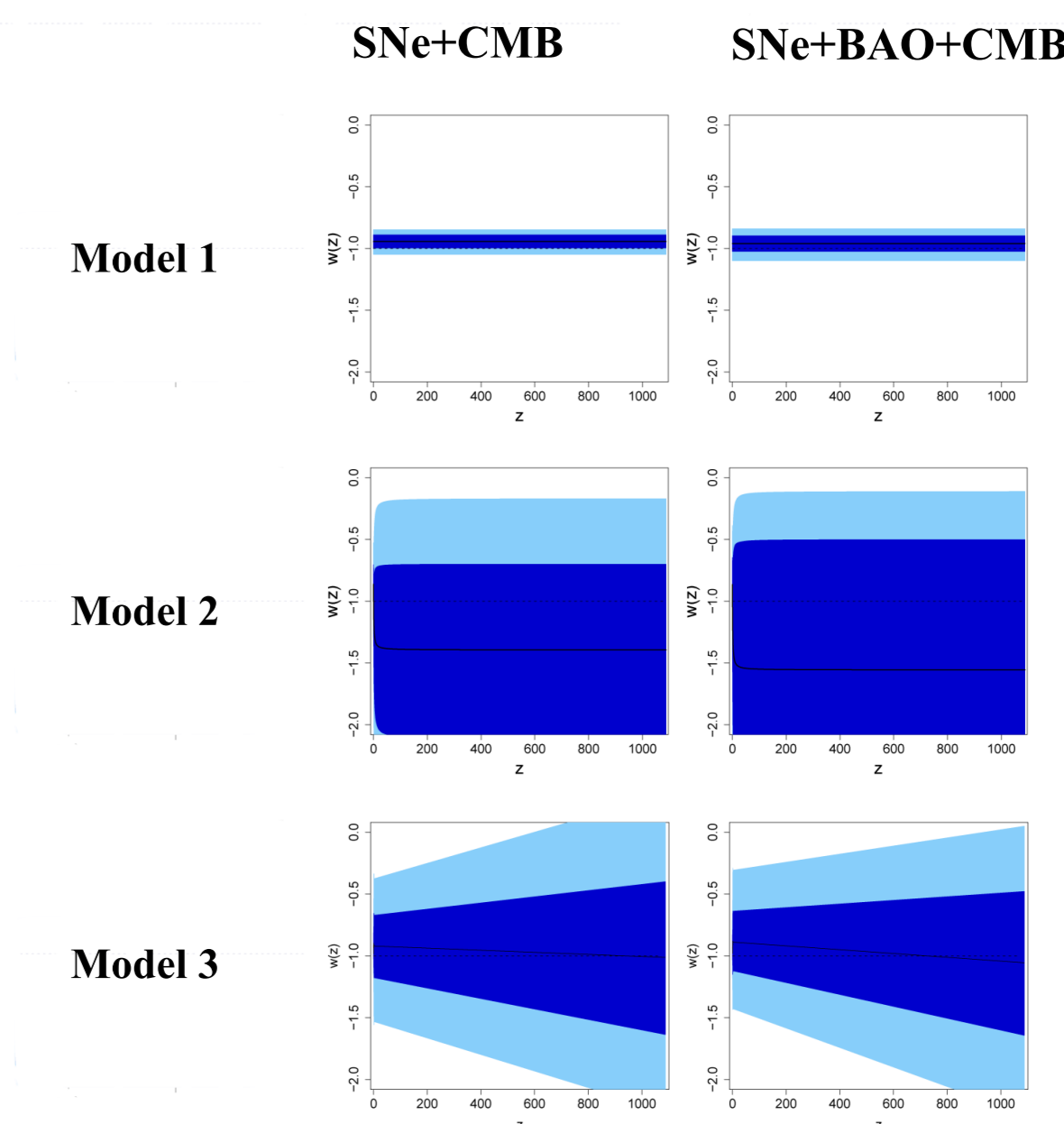
The specific priors for this model will be: $\pi(\kappa^2) \sim IG(25, 9)$ and $\pi(\rho) \sim Be(6, 1)$

Results

	a	b	Ω_m	$\Omega_r 10^{-5}$	H_0	M	σ^2
Model 1							
SNe	(-1.117,-0.800)	NA	(0.219,0.308)	(4.1,5.6)	(66.38,77.15)	(0.03,0.37)	(1.02,1.34)
SNe+BAO	(-1.163,-0.787)	NA	(0.230,0.320)	(4.2,6.0)	(64.26,76.42)	(-0.04,0.35)	(1.03,1.37)
SNe+CMB	(-1.048,-0.848)	NA	(0.233,0.292)	(4.3,5.5)	(67.25,75.42)	(0.07,0.32)	(1.02,1.34)
SNe+BAO+CMB	(-1.099,-0.840)	NA	(0.239,0.303)	(4.3,5.6)	(66.28,75.50)	(0.04,0.31)	(1.03,1.35)
Model 2							
SNe	(-1.270,-0.469)	(-2.000,3.486)	(0.223,0.317)	(4.2,5.7)	(65.98,76.61)	(0.03,0.36)	(1.03,1.34)
SNe+BAO	(-1.538,-0.233)	(-3.514,5.518)	(0.236,0.328)	(4.3,6.2)	(63.35,75.40)	(-0.05,0.32)	(1.03,1.36)
SNe+CMB	(-1.174,-0.493)	(-0.975,2.576)	(0.230,0.301)	(4.3,5.5)	(66.94,75.95)	(0.05,0.34)	(1.02,1.35)
SNe+BAO+CMB	(-1.218,-0.318)	(-1.090,4.263)	(0.239,0.319)	(4.3,5.9)	(64.64,75.84)	(-0.01,0.34)	(1.03,1.37)
Model 3							
SNe	NA	NA	(0.221,0.313)	(4.2,5.6)	(66.13,76.76)	(0.02,0.37)	(1.02,1.34)
SNe+BAO	NA	NA	(0.240,0.301)	(4.4,5.5)	(66.79,74.64)	(0.05,0.29)	(1.02,1.34)
SNe+CMB	NA	NA	(0.233,0.296)	(4.3,5.4)	(67.40,75.62)	(0.07,0.33)	(1.02,1.34)
SNe+BAO+CMB	NA	NA	(0.242,0.294)	(4.4,5.5)	(67.14,74.87)	(0.07,0.30)	(1.02,1.34)



Larger Range z=[0,2000]



Conclusions

- The non-parametric model provides the most flexible fit for the dark energy equation of state without guessing at a form of $w(z)$
- The non-parametric model provides tighter probability bands and is able to be coherent beyond $z = 2$ unlike the favored parametric model
- Adding CMB data provides tighter probability bands and a better fit even on $z=[0,2]$ range

Future Work

- We would like to explore other priors for H_0 and Ω_m
- We would like to continue to add more SNe, BAO, and CMB data as it becomes available to further explore dark energy equation of state
- We would like to do an experimental design to see where more data on the z axis is most beneficial to further constrain the dark energy equation of state

References

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